PROFESSOR RUSSELL'S INFINITE.

In *Scientific Method in Philosophy,* p. 159, Prof. Bertrand Russell says, "In the present lecture I wish to state and explain the problem of infinity, to show how it arose, and to show the irrelevance of all the solutions proposed by philosophers."

Again, p. 179, "The difficulties of infinity are of two kinds, of which the first may be called sham, while the others involve, for their solution, a certain amount of new and not altogether easy thinking. The sham difficulties are those suggested by the etymology, and those suggested by the confusion of the mathematical infinite with what philosophers impertinently call the true infinite."

Professor Russell says the infinite is a problem in philosophy. But philosophers have been unequal to their problem. They are "impertinent" when they do not accept the work of mathematicians in the field of philosophy. As I understand Professor Russell, we get the infinite in a series and in a certain kind of number. Failure to accept the infinite as a mathematical series is impertinent.

I have no desire to be "impertinent," but I must admit that Professor Russell's infinite is not impressive. 2/7 involves a series, 285714, 285714, 285714. This is a series that repeats itself in six terms. That is, the process of dividing two by seven cannot be completed. As far as we go in the dividing, the results are clear, exact, correct, and finite. Manifestly the 2/7 is not infinite. It is less than one. The dividing is not infinite. It is simple, clear, and correct. The series is a six-term series. It is different from a one-term series. The six terms are not infinite. They are old friends, quite finite. And two or several of them will not be infinite. The mystery lies in the repetition.

If one insists that a fraction of 1 is infinite, I know of no mode of stopping him. But this much is clear. Before we can proceed satisfactorily, we must agree upon our term. What are we to understand by infinite?

Infinite is one of a verbal trinity used to designate reality. Reality is infinite, absolute, universal. These words mark different functions of reality and have been uncovered by separate lines of intelligent activity. As I understand, we owe the knowledge that reality is infinite to mathematics. Mathematics is the "look-out" on the bridge. It reports the infinite in the offing. It tells us nothing

of the crew, the cargo, or the life history of the craft. What is the content of the word infinite? What does it tell us of reality?

Let us consider. Take a stick an inch long. This stick is wood. But wood is not stick. Stick is more than wood. It is wood and two ends. The ends are not wood. And yet the stick cannot be without the ends. What are the ends? The end is the complete absence of wood. A stick is wood and then the wood must cease. Both are essential. And neither is in any sense the other. Ends are not sticks. Wood is not stick. We may say the stick is wood, wood interrupted or broken. But there is no wood at all in the end. Nor can the wood be the cause of the end. In no sense is end wood, or the wood end. Yet both are necessary to the stick. That is, the stick of wood depends upon something that is not wood, that has nothing in common with wood, that lies outside and beyond wood. This is what we understand by finite. Finiteness is dependence. Dependence is that which has its existence, its *Dasein,* outside itself. It is what it is because of its other. And its other lies over against it, beyond it. The end of the stick does not enter into the stick. It is outside. And it differentiates the stick. That is, the stick as stick, as length, depends upon the end. That is, it is what it is because of its outsideness. The difference between a foot-stick and a mile-stick is in the ends. That which is not the thing, the outside, becomes important. Outsideness may be place, distance, or form.

If I offer a sample of distance, can you tell whether it is taken from the inch, the mile, or the ten miles? Outsideness has no quality. There are no marks that differentiate it. Place, distance, form are indifferent to their content. The difference is no part of the space. The space in an inch is exactly the space in a mile. We do not break it or cut it or end it. The object occupying the space is moving along the distance that is stopped. If the moving body is stopped here, it is an inch; there, it is a mile. The space on one side of the moving point is exactly like the space on the other side. We do not stop the space, but the moving body. That is, we have in space a case where the moment of negation is absorbed. Space is not affected by change, by ends, or differences.

The finite we saw to be that which is dependent upon its opposite for its *Dasein.* The infinite is that which is not subject to change, not affected by it at all. In the infinite the difference or change is *aufgehoben,* contained as a moment.

When the moment of negation ceases as difference, we get beyond the finite. We have the infinite. Space uncovers this function of reality. It was the merit of the geometricians to get this vision. It was a master vision and has brought wide service to man and mathematics.

The infinite is not any definite space, but space as not affected by interruption. Space does not absorb the interruption. It maintains itself beyond the reach of interruption. That is, as Hegel would say, space negates the moment of negation. The interruptions remain to carry on their business. Hence to set up the infinite as full reality is to practise idolatry. What we wish is an infinite that not only resists change but absorbs all change. An infinite in which change is a moment of itself, ceases to be in any sense different. Here we uncover the basis for the true and the false infinite.

Let us return to our example, 2/7. This is called an infinite series. The 2/7 is manifestly not infinite. It is dependent upon one and is less than one. The division is not infinite. It is definite, clear, correct. We get 285714, 285714, and so on as long as our patience and industry sustain us. It is a six-term series. If we take 4/14, we have the same situation. The mystery is not the 2/7, nor the division. It is rather that which the division cannot reach. Try as we may, it eludes us. That is, it is not affected by the process. It is beyond the reach of the process. It lures the process of division, and yet is utterly beyond it. It maintains itself permanently beyond the process. To this extent the process is negated. But the process of division is not absorbed. The two stand over against each other.

We reach this conclusion: The 2/7 are not infinite; the six terms are not infinite; yet the infinite is present compelling the repetition, yet remaining beyond its reach.

The case is this way: Reality is infinite. The humble fraction, 2/7, shows the infinite. This discovery we owe to mathematics. This is a high service to philosophy. Philosophers are appreciative and publicly give thanks.

Columbus discovered America, and returned to Spain to die. Is it "impertinent" in us to explore his discovery and unfold its riches? We find the infinite in mathematics. That stands. But the infinite of mathematics is a lonely gentleman. He takes no part

even in the process that uncovers him. Verily "His ways are not like our ways."

It is true that we have a chapter of civilization built upon this infinite of geometry, a static stage marked by permanent separations of classes. But it is equally true that civilization has passed beyond this stage. And so the infinite has a career. There is an evolution. Reality is infinite; reality is also absolute. Further, reality is universal. And so it appears that philosophy has a task of its own. Philosophy is not ungrateful to mathematics. But it regards the work of mathematics as quite preliminary, hardly more than that of Columbus, so far as its own problem is concerned.

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INFINITY AND THE PART-AND-WHOLE AXIOM.

DEFINITIONS OF THE FUNDAMENTAL ENTITIES OF GEOMETRY.[1]

The definition of *infinity* as endlessness, or as a quantity that is greater than any assignable finite quantity, is given in all the text-books on elementary mathematics with which I am familiar, and, so far as I know, was the only one used in this science until Cantor introduced his theory of transfinite numbers. Thus defined, the term is perfectly intelligible and in accord with common sense, because every intelligent person is familiar with the fact that every magnitude is divisible, at least mentally, into an endless number of parts, or can be increased to any other magnitude by adding to it other magnitudes of the same kind, and the human mind cannot conceive how either of these processes could be brought to an end, even if it were continued to all eternity.

Modern mathematicians, however, claim to have reasons for being dissatisfied with this simple definition, and are defining infinity in a manner that not only is not simple and perfectly intelligible, but, so far as I can see, violates Euclid's ninth axiom which, in accord with common sense, proclaims that the whole is greater than its part. Thus, the author of the article on "Number" in the *Encyclopædia Britannica* (11th edition, Vol. XIX, p. 847), defines

[1] The material of this article will be embodied in a chapter of a book on "Science, Truth, Religion, and Ethics" which I am preparing for publication.